Assigned: Friday, December 7
Due: Wednesday, December 12
Note: Please hand in problems on separate sheets of paper

Problem S9.1: Look-back to Lectures S15, S16, S17 (10 points)

Is Lecture S10, we derived a state-space model for attitude control of a satellite. Here we will consider only the control input, F_c, giving the model

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{d}{J} \end{bmatrix}}_{B} F_c \\ y &= \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (D = 0) \end{aligned}$$

Use the values $d/J = 1$.

(a) Using the state transition matrix, compute (by hand) the state response of the system to a step input, $u(t) = \bar{F}$, $t > 0$ (where $\bar{F}$ is a constant). The satellite is initially at rest, i.e. $\vec{x}(0) = [0\ 0]^T$.

(b) For the model considered in class, only state x_1 was measured in the output equation. Here we will assume that both states x_1 and x_2 are available for use in a feedback controller. Design (by hand) a full-state feedback controller that places the eigenvalues of the controlled system at $s = -1 \pm j$.

(c) Draw the block diagrams of your uncontrolled and controlled systems.

Problem S9.2: Look-back to Lectures S17, S18 (10 points)

A motor position servo has state-space model

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 30 \\ 0 & 0 & -6 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} u$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{D} u,$$

where the states are $x_1 = \theta$, the shaft position, $x_2 = \dot{\theta}$, the shaft velocity, and $x_3 = i_f$, the field current. The input is $u(t) = e_f(t)$, the applied field voltage. The output equation tells us that all three states are measured as outputs.

(a) Compute (by hand) the eigenvalues of the motor position servo system.

(b) Design (by hand) a full-state feedback controller that places the eigenvalues of the controlled system at $s = -7$ and $s = -3 \pm 3j$.

(c) Draw the block diagrams of your uncontrolled and controlled systems. You can draw just the state equations (you do not need to draw the part of the diagram for the output equation).